



# SSV case 2

---

*EE4*

**Team Synnove - PM7**

*Boogaerts Jisca*

*Dejans Arnout*

*Dendal Rik*

*Ecker Michiel*

*Geelen Gertjan*

*Van Tricht Arnout*

*10-5-2013*

---

## Table of contents

Table of contents .....	1
I. Correction of the Sankey diagram .....	1
At maximum velocity .....	1
At half of the maximum velocity (on slope) .....	2
II. Analysis of the drive shaft .....	4
III. 2D technical drawing .....	12
IV. Collision .....	15
V. Exercise: biker.....	16

## I. Correction of the Sankey diagram

For the first diagram we used some theoretical parameters, which we can now replace by the experimentally determined values of our SSV. We measured the weight and the frontal area of the car and we calculated the correct Crr. We also got a new solar panel, which means we had to do new measurements for the new diode factor.

### At maximum velocity

#### Start

The new solar panel has other dimensions, this means we start from another amount of incoming energy.

$$P = 800 \frac{W}{m^2} * 0,180m * 0,360m = 51.84W$$

#### Loss of the solar panel

Using the bisection method we solve the following equation.

$$I = I_{sc} - I_s \left( e^{\frac{E+RI}{m*N*U_r}} - 1 \right)$$

With  $E=4,745$  (calculated in the first version of the Sankey diagram)  
 $I_{sc}=0.88A$   
 $I_s=10^{-8}A$   
 $m=1,216$  (new diode factor)  
 $N=16$   
 $U_r=0,0257V$   
 $R=3,32\Omega$

We find  $I=1,326A$

$$U = I * R + E = 1,326 * 3,32 + 4,745 = 9,147V$$

Knowing  $I$  and  $U$  we can calculate the output power of the solar panel.

$$P_{remaining} = U * I = 1,326 * 9,147 = 12,13W$$

#### Loss of the motor

The losses in the motor depend on the internal resistance and the current through it. With this information given, we can calculate the power losses in the motor.

$$P = I^2 * R = 1,326 * 3,32 = 5,836W$$
$$P_{remaining} = 6,294 W$$

### Loss air resistance

The first assumption of the frontal area was correct. The  $F_{air}$  will be the same as in the first Sankey diagram.

$$F_{air} = \frac{\rho \cdot A \cdot C_w \cdot v^2}{2} = 0,0885$$

$$P_{loss\ air} = F_{air} \cdot v = 0,327W$$

$$P = 6,294 - 0,327 = 5,967\ W$$

### Loss rolling resistance and transmission

For determining the rolling resistance of the car, we placed the SSV on top of the race track, and let it roll down the hill. It rolled 12 m. By using law of conservation of energy we calculated the rolling resistance. The potential energy on top of the hill is equal to the energy that the car loses because of the rolling resistance, the friction and the air resistance.

The velocity used in this calculation is the average velocity during the rolling down on the track.

$$m \cdot g \cdot h = (m \cdot g \cdot C_{rr} \cdot x) + \frac{\rho \cdot A \cdot C_w \cdot v^2}{2} \cdot x$$

$$0,800 \cdot 9,81 \cdot 0,5 = (0,800 \cdot 9,81 \cdot C_{rr} \cdot x) + \frac{1,293 \cdot 0,2 \cdot 0,5 \cdot 1,09^2}{2} \cdot x$$

$$3,92 = (0,800 \cdot 9,81 \cdot C_{rr} \cdot 12)$$

$$C_{rr} = 0,038$$

This value for  $C_{rr}$  also contains the friction of the transmission. The power losses due to this  $C_{rr}$  are:

$$P = m \cdot g \cdot C_{rr} \cdot v = 1,10\ W$$

$$P_{remaining} = 5,867\ W$$

### At half of the maximum velocity (on slope)

#### Start

The new solar panel has other dimensions, this means we start from another value.

$$P = 800 \frac{W}{m^2} \cdot 0,180m \cdot 0,360m = 51.84W$$

### Loss of the solar panel

Using the bisection method we solve the following equation.

$$I = I_{sc} - I_s \left( e^{\frac{E + RI}{m \cdot N \cdot U_r}} - 1 \right)$$



With  $E=3,954$  (calculation see first Sankey diagram)  
 $I_{sc}=0.88A$   
 $I_s=10^{-8}A$   
 $m=1,216$  (new diode factor)  
 $N=16$   
 $U_r=0,0257V$   
 $R=3,32\Omega$

We find  $I=1,564A$

$$U = I \cdot R + E = 1,326 \cdot 3,32 + 3.954 = 9,147V$$

Knowing  $I$  and  $U$  we can calculate the power that the solar panel gives.

$$P = U \cdot I = 1,564 \cdot 9,147 = 14,306W$$

### *Loss of the motor*

$$P = I^2 \cdot R = 1,564 \cdot 3,32 = 8,122W$$

$$P_{remaining} = 6,184 W$$

### *Loss air resistance*

$$F_{air} = \frac{\rho \cdot A \cdot C_w \cdot v^2}{2} = 0,0221$$

$$P_{loss air} = F_{air} \cdot v = 0,041W$$

$$P_{remaining} = 6,184 - 0,041 = 6,143 W$$

### *Loss rolling resistance and transmission*

$$P_{remaining} = 6,143 - m \cdot g \cdot C_{rr} \cdot v = 5,63 W$$

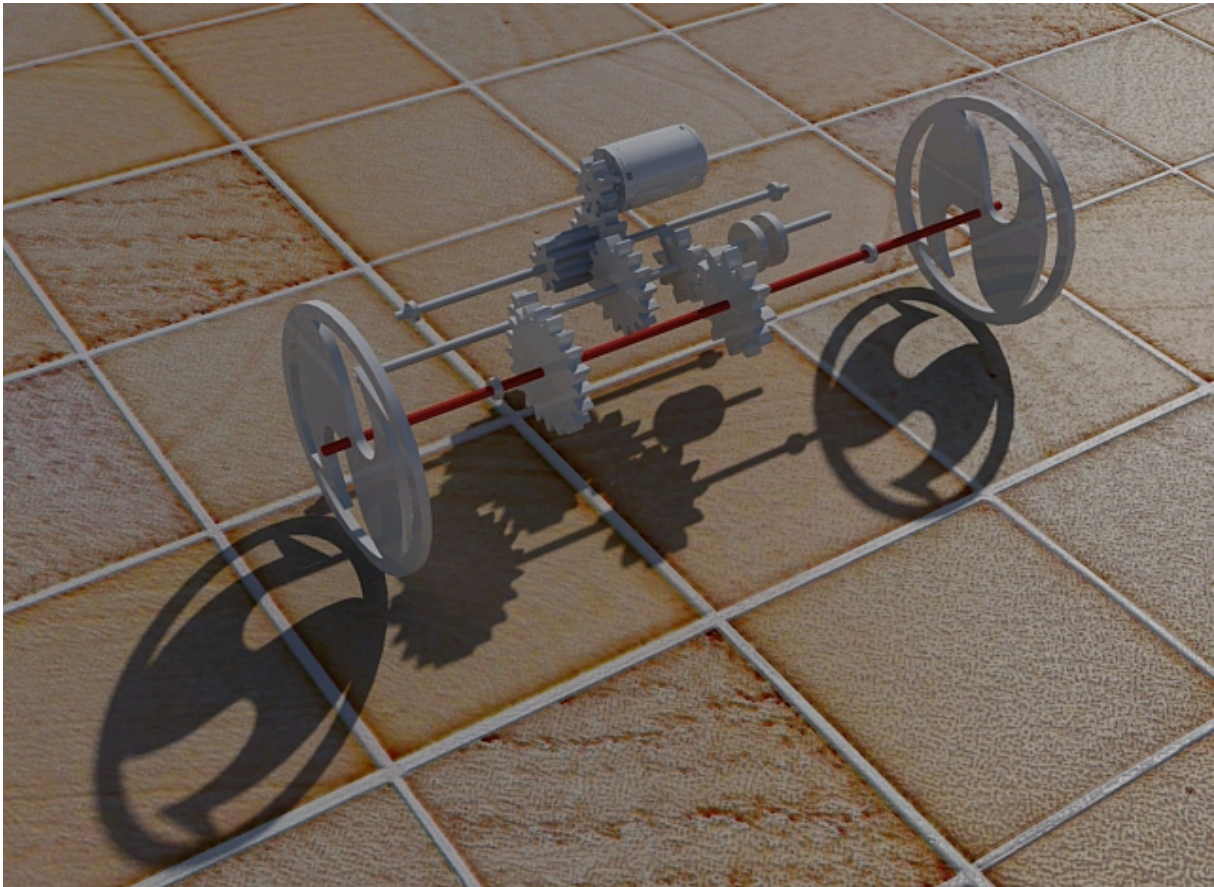
### *Loss slope*

$$F = \sin\left(\arctan\left(\frac{1}{8}\right)\right) \cdot 0,8 \cdot 9,81 \cdot 1,85 = 1,801$$

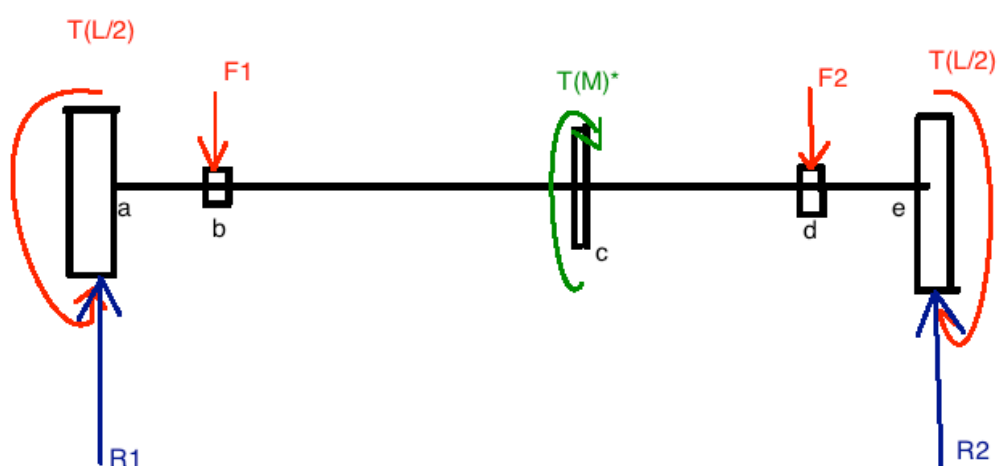
$$P = 1,801 \cdot 1,85 = 3,33 W$$

$$P_{remaining} = 2,3 W$$

## II. Analysis of the drive shaft



*This is a 3D drawing of our drive shaft used during the calculations. Made in AutoCAD*



*We are only showing the gear used in these calculations on the schematic representation above.*

## Situation 1:

The SSV accelerates from standstill.

First we'll calculate the Shear force and Bending moment. This was done in maple:

$\Sigma F=0$

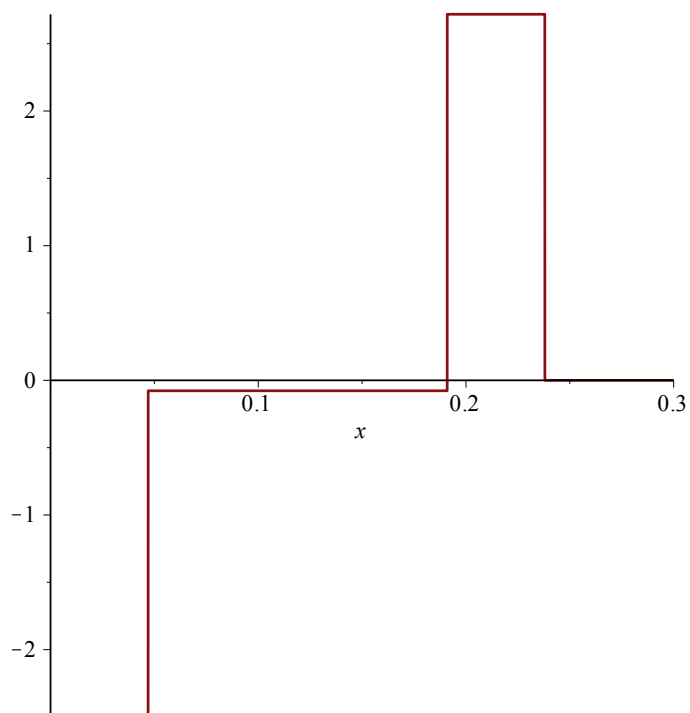
$\Sigma M=0$

We will be using T for the moment of inertia in our calculations.

```

> restart;
> SF:=R1+R2-N2-N1=0;
                                 $SF := R1 + R2 - N2 - N1 = 0$  (1)
> N1:=2.796;
                                 $N1 := 2.796$  (2)
> N2:=2.403;
                                 $N2 := 2.403$  (3)
> SM:=N2*0.047+N1*0.191-R2*0.238=0;
                                 $SM := 0.646977 - 0.238 R2 = 0$  (4)
> solve(SM,R2);
                                2.718390756 (5)
> R2:=%;
                                 $R2 := 2.718390756$  (6)
> solve(SF,R1);
                                2.480609244 (7)
> R1:=%;
                                 $R1 := 2.480609244$  (8)
> u := x -> piecewise( x< 0 , 0 , x >= 0 ,1);
                                 $u := x \rightarrow \text{piecewise}(x < 0, 0, 0 \leq x, 1)$  (9)
> fD:=(-R1*(u(x)))+(N2*(u(x-0.047)))+(N1*(u(x-0.191)))-(R2*u(x-0.238))
; Distances from "a" to "b", "b" to "d" and "d" to "e" (see added schematic representation of the
drive shaft.
 $fD := -2.480609244 + 2.403 \left( \begin{cases} 0 & x < 0.047 \\ 1 & 0 \leq x - 0.047 \end{cases} \right) + 2.796 \left( \begin{cases} 0 & x < 0.191 \\ 1 & 0 \leq x - 0.191 \end{cases} \right)$  (10)
                                 $- 2.718390756 \left( \begin{cases} 0 & x < 0.238 \\ 1 & 0 \leq x - 0.238 \end{cases} \right)$ 
> plot(fD,x=0..0.3);

```

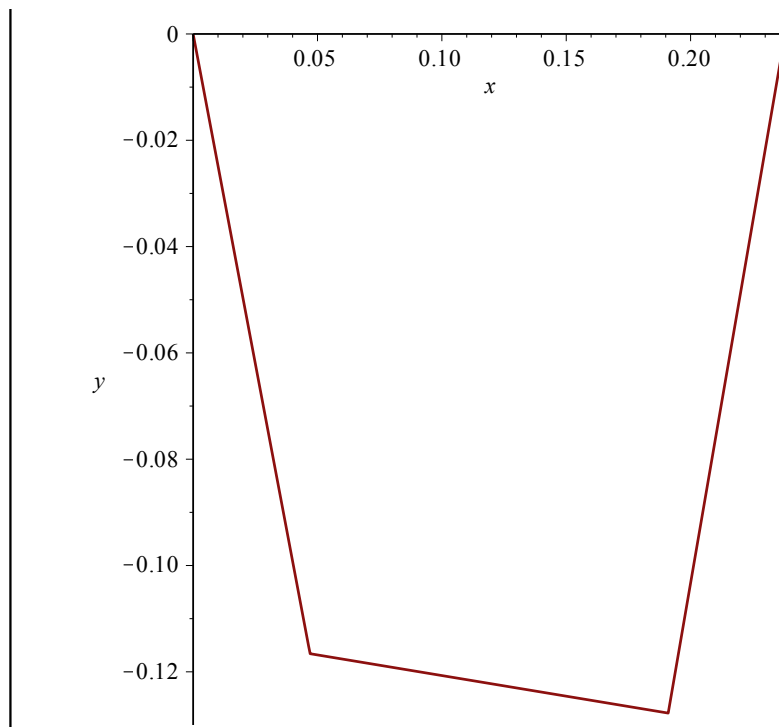


```
> M:=(-R1*(x)*(u(x)))+N2*(x-0.047)*u(x-0.047)+N1*(x-0.191)*u(x-0.191)
; Distances from "a" to "b", "b" to "d" and "d" to "e" (see added schematic representation of the
drive shaft.
```

$$M := -2.480609244 x \begin{pmatrix} 0 & x < 0 \\ 1 & 0 \leq x \end{pmatrix} + 2.403 (x - 0.047) \begin{pmatrix} 0 & x < 0.047 \\ 1 & 0 \leq x - 0.047 \end{pmatrix} + 2.796 (x - 0.191) \begin{pmatrix} 0 & x < 0.191 \\ 1 & 0 \leq x - 0.191 \end{pmatrix} \quad (11)$$

```
> plot(M,x=0..0.240,y=0..-0.13);
```





Bending moment: This will be greatest at that point where the moment is largest. So we'll calculate it at this point. At  $x=0.191$ .

$$\sigma := \frac{M_{max} y}{T} \quad (12)$$

$$M_{max} := (-R1 * (0.191)) + N2 * (0.191 - 0.047) + N1 * (0.191 - 0.191);$$

$$M_{max} := -0.1277643656 \quad (13)$$

$$y := 0.001; \text{ // This is half the diameter of our axle.}$$

$$y := 0.001 \quad (14)$$

$$T := (\pi * d^4) / 64;$$

$$T := \frac{1}{64} \pi d^4 \quad (15)$$

$$d := 0.002;$$

$$d := 0.002 \quad (16)$$

$$\sigma := \frac{-5.110574624 \cdot 10^8}{\pi} \quad (17)$$

$$\sigma := \text{simplify}(\sigma); \text{ // MPa or N/m}^2$$

$$\sigma := -1.626746426 \cdot 10^8 \quad (18)$$

The minimum tensile strength is about 400 MPa so almost 3 times as high.

The maximum bend is about 163MPa, which is a lot less than the shaft can handle. Additionally, the shaft will only be subjected to this bend for a very short period of time.

The next maple file shows the calculations of the Torsion:

We'll be using  $T_p$  instead of  $I_p$  as polar moment of inertia because maple sees  $I$  as the imaginary number.  $T_{mot2}$  is  $T_{mot}*$ , meaning the moment from the motor, adjusted by the transmission, on the axle we are investigating now.  
 $rtr$  and  $ntr$  stand for, respectively, the transmission ratio and the transmission loss.

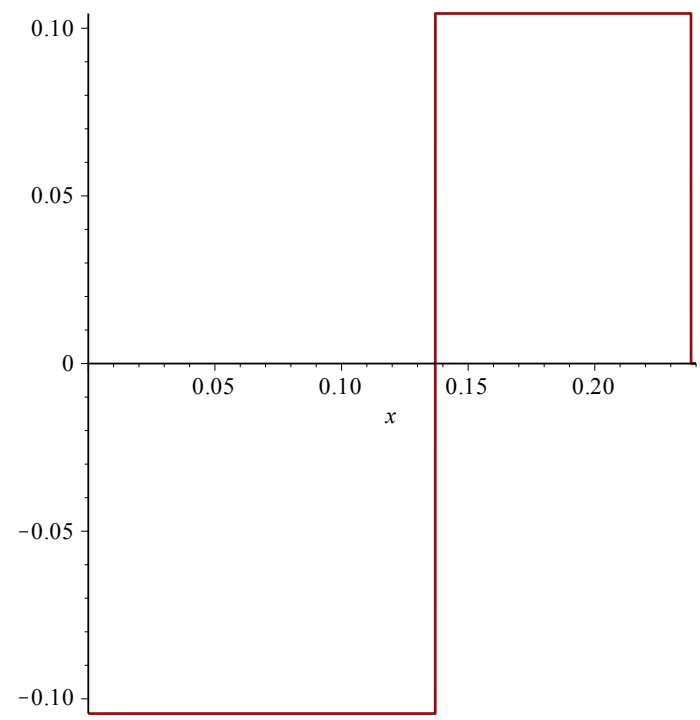
```

> restart;
> m:=0.030; //(kg)
                                      $m := 0.030$  (1)
> rwheel:=0.04; //(m)
                                      $rwheel := 0.04$  (2)
> a:=1.449; //(m/s^2)
                                      $a := 1.449$  (3)
> Tmot:=0.0232; //(Nm)
                                      $Tmot := 0.0232$  (4)
> Jmot:=4.10*10^(-7); //(kg*m^2)
                                      $Jmot := 4.100000000 \cdot 10^{-7}$  (5)
> rtr:=1/10;
                                      $rtr := \frac{1}{10}$  (6)
> ntr:=0.9;
                                      $ntr := 0.9$  (7)
> VGL:=Tmot2-Tload= ((Jmot*ntr) / (rtr^2)+Jload)*alphaload;
                                      $VGL := Tmot2 - Tload = (0.00003690000000 + Jload) alphaload$  (8)
> Tmot2:=ntr*Tmot/rtr;
                                      $Tmot2 := 0.20880$  (9)
> Jload:=m*rwheel^2;
                                      $Jload := 0.0000480$  (10)
> alphaload:=a*rtr;
                                      $alphaload := 0.1449000000$  (11)
> solve(VGL,Tload);
                                      $0.2087876980$  (12)
> Tload:=%;
                                      $Tload := 0.2087876980$  (13)
> u := x -> piecewise( x< 0 , 0 , x >= 0 ,1);
                                      $u := x \rightarrow piecewise(x < 0, 0, 0 \leq x, 1)$  (14)
> T:=((-Tload/2)*(u(x)))+(Tmot2*(u(x-0.137)))-((Tload/2)*(u(x-0.238)));
                                     (15)

$$T := -0.1043938490 \left( \begin{cases} 0 & x < 0 \\ 1 & 0 \leq x \end{cases} \right) + 0.20880 \left( \begin{cases} 0 & x < 0.137 \\ 1 & 0 \leq x - 0.137 \end{cases} \right)$$


```

```

-0.1043938490  $\left( \begin{cases} 0 & x < 0.238 \\ 1 & 0 \leq x - 0.238 \end{cases} \right)$ 
> plot(T,x=0..0.240);

> Tmax:=Tmot2-Tload/2;
Tmax := 0.1044061510 (16)
> Torsion:=(Tmax*rshaft)/Tp;
Torsion :=  $\frac{0.1044061510 \text{ rshaft}}{Tp}$  (17)
> rshaft:=0.001; //(m)
rshaft := 0.001 (18)
> Tp:=(Pi*d^4)/32;
Tp :=  $\frac{1}{32} \pi d^4$  (19)
> d:=0.002;
d := 0.002 (20)
> Torsion;
 $\frac{2.088123020 \cdot 10^8}{\pi}$  (21)
> Torsion:=simplify(%);
Torsion := 6.646702007 107 (22)

```

## Situation 2.

In the next situation, the SSV is at maximum speed, so the shaft will be subjected to a smaller Torque. There's no need to redo the shear force and bend moment calculations because they're the same in both situations. The Torsion will be different though:

```
[> restart;
```

Second situation: the SSV is at full speed, so the Torsion will be less than when just speeding up.  
 $T_{mot} \cdot T_{load} = ((J_{mot} \cdot \eta_{tr} / r_{tr}^2) + J_{load}) \cdot \alpha_{load}$ .  
This is the formula to describes the Torque of our SSV. Here we see an acceleration ( $\alpha$ ). Our SSV is at top speed, so  $\alpha$ last equals zero:  
 $T_{mot} = T_{load}$

```
> Tload:=Frr*rwheel;                                Tload := Frr rwheel                                (1)
> Frr:=Crr*N;                                         Frr := Crr N                                (2)
> N:=m*g;                                             N := m g                                    (3)
> g:=9.81;                                           g := 9.81                                    (4)
> rwheel:=0.04;                                       rwheel := 0.04                              (5)
> rwheelf:=0.02;                                      rwheelf := 0.02                             (6)
> Crr:=0.0304;                                        Crr := 0.0304                               (7)
```

We have three wheels, so  $T_{last}$  equals the sum of the rolling resistance of these three wheels. We'll name them  $T_l$ ,  $T_r$  and  $T_f$ . (l=left, r=right, f=front).

```
> Tl:=Crr*ml*g*rwheel;                                Tl := 0.01192896 ml                            (8)
> ml:=0.285;                                          ml := 0.285                                    (9)
> Tr:=Crr*mr*g*rwheel;                                Tr := 0.01192896 mr                            (10)
> mr:=0.245;                                          mr := 0.245                                    (11)
> mf:=0.255;                                          mf := 0.255                                    (12)
> Tf:=Crr*mf*g*rwheelf;                              Tf := 0.00152094240                            (13)
> Tload:=Tl+Tr+Tf;                                    Tload := 0.00784329120                        (14)
> Tmot:=Tload;                                        Tmot := 0.00784329120                        (15)
```

For the calculation of the Torsion on our shaft, we'll divide the delivered torque for  $T_{last}$  evenly over the two wheels on the shaft.

```
> u := x -> piecewise( x< 0 , 0 , x >= 0 ,1);
u := x→piecewise(x < 0, 0, 0 ≤ x, 1)                                (16)
```



```

> T:=((-Tload/2)*(u(x)))+(Tmot*(u(x-0.137)))-((Tload/2)*(u(x-0.238)))
;

$$T := -0.003921645600 \begin{pmatrix} 0 & x < 0 \\ 1 & 0 \leq x \end{pmatrix} + 0.00784329120 \begin{pmatrix} 0 & x < 0.137 \\ 1 & 0 \leq x - 0.137 \end{pmatrix} - 0.003921645600 \begin{pmatrix} 0 & x < 0.238 \\ 1 & 0 \leq x - 0.238 \end{pmatrix} \quad (17)$$

> plot(T,x=0..0.240);

```

```

> Torsion:=(Tmax*rshaft)/Tp;

$$Torsion := \frac{T_{max} r_{shaft}}{T_p} \quad (18)$$

> Tmax:=Tload/2;

$$T_{max} := 0.003921645600 \quad (19)$$

> rshaft:=0.001;

$$r_{shaft} := 0.001 \quad (20)$$

> Tp:=(Pi*d^4)/32;

$$T_p := \frac{1}{32} \pi d^4 \quad (21)$$

> d:=0.002;

$$d := 0.002 \quad (22)$$

> Torsion;

$$\frac{7.843291200 \cdot 10^6}{\pi} \quad (23)$$

> Torsion:=simplify(%);

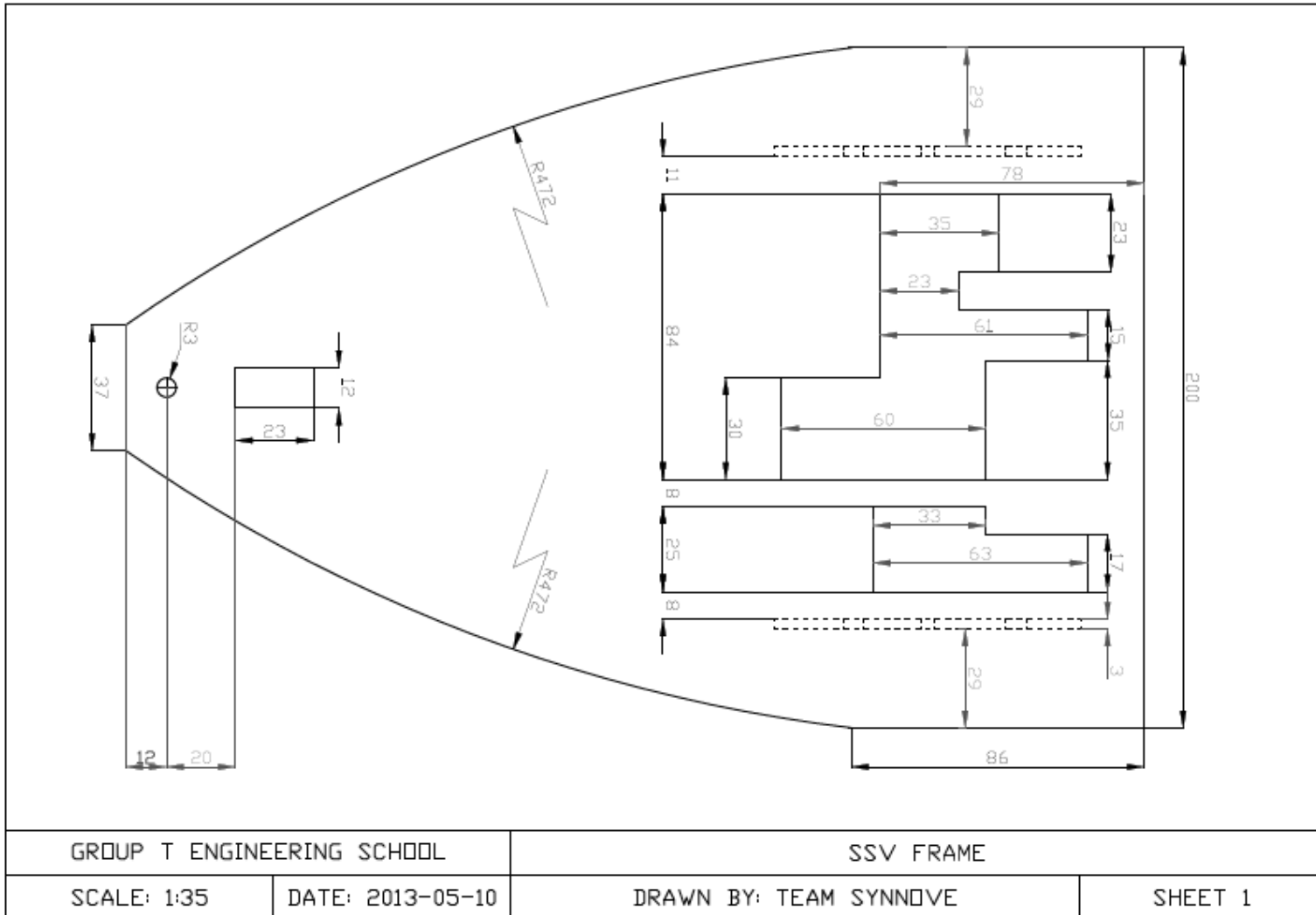
$$Torsion := 2.496597129 \cdot 10^6 \quad (24)$$


```

From these calculations we can see that the Torsion is almost 30 times smaller at top speed as compared to when the SSV is just starting to accelerate.

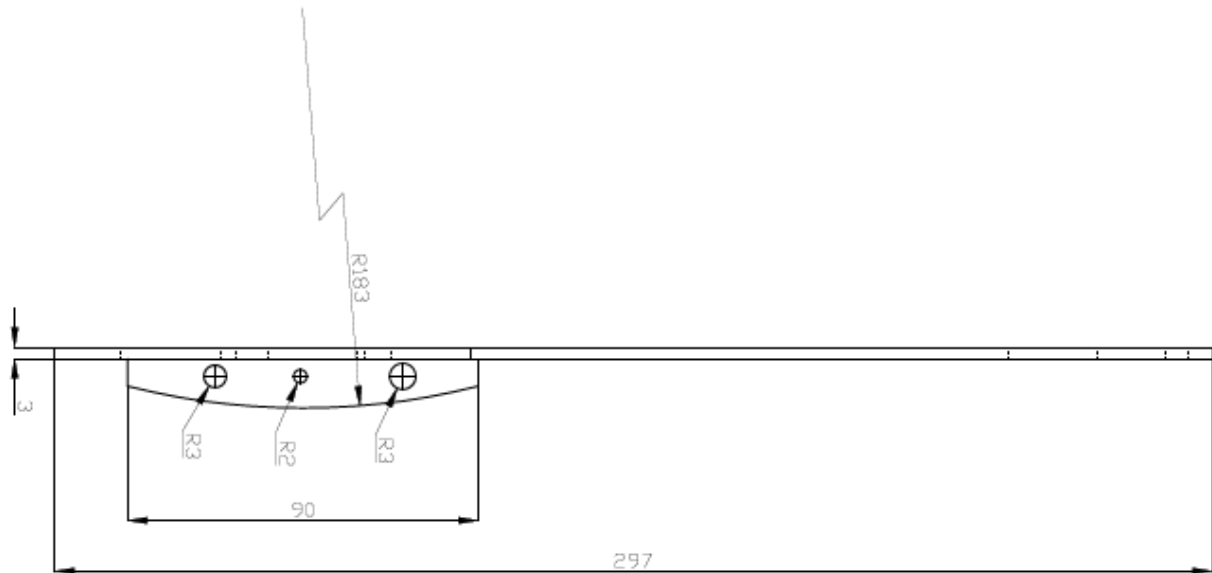
### III. 2D technical drawing

Top-view:



Technical drawing of the top view.

Side view:

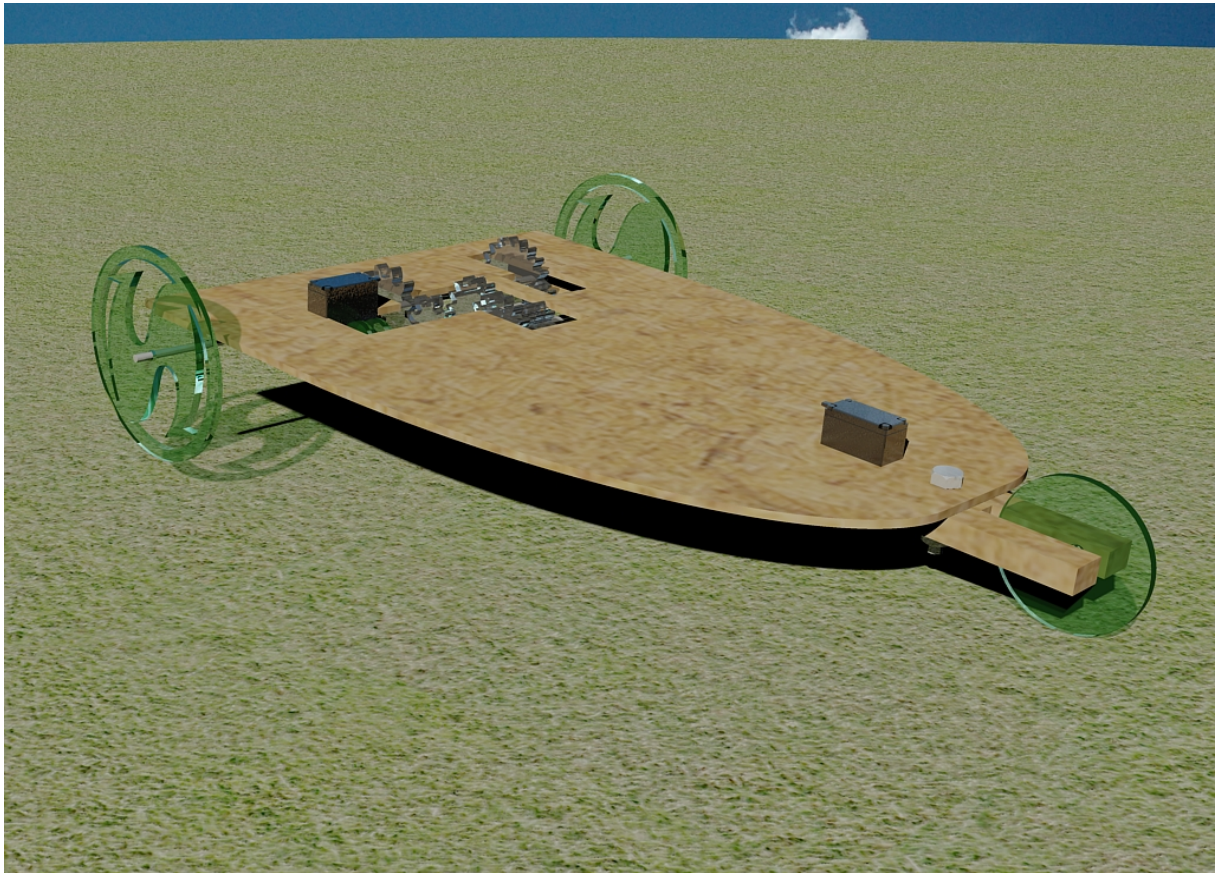


GROUP T ENGINEERING SCHOOL		SSV FRAME	
SCALE: 1:35	DATE: 2013-05-10	DRAWN BY: TEAM SYNNOVE	SHEET 2

*Technical drawing of the side view.*

When either of the sketches above isn't clear enough, you can see the PDF file on our Wikiversity.

We also made some 3D drawings of our small solar vehicle:



*3D drawing made in AutoCAD, top view, without solar panel.*



*3D drawing made in AutoCAD, side view, with solar panel.*



## IV. Collision

*Given:*

$$V_A = 3.7 \text{ m/s}$$

$$m_A = 1.2 \text{ kg}$$

$$e=1$$

**Asked:**  $\Delta t = ?$

*Solution:*

$V_{A,c}$  = the speed on the collision axle before the collision

$$V_{A,c} = 3.7 \sin 16^\circ = -0.64 \text{ m/s}$$

$V_{A,c}'$  = the speed on the collision axle after the collision

$e=1$  because it is an elastic collision

$$e = \frac{-(V_{B,c}' - V_{A,c}')}{V_{B,c} - V_{A,c}} = 1$$

$V_{B,c} = V_{B,c}' = 0$  because the body B is the wall here and it doesn't move before or after the collision.

If we fill in these values:

$$1 = \frac{-(0 - V_{A,c}')}{0 - (-0.64)}$$

we find a  $V_{A,c}' = 0.64 \text{ m/s}$ .

If we make the body free:

$$L_{A,c}' - L_{A,c} = \int_0^t \langle F \rangle dt$$

$$\langle F \rangle = 10$$

The impulse is  $L_{A,c} = m \cdot V_{A,c} = 1.2 \cdot 0.64 = 0.768 \frac{\text{kg} \cdot \text{m}}{\text{s}}$

If we fill in everything in the equation, we find:  $1.2 \cdot (0.64 - (-0.64)) = 10 \cdot t$

So we find that  $\Delta t$  should be at least 0.154 seconds.

## V. Exercise: biker

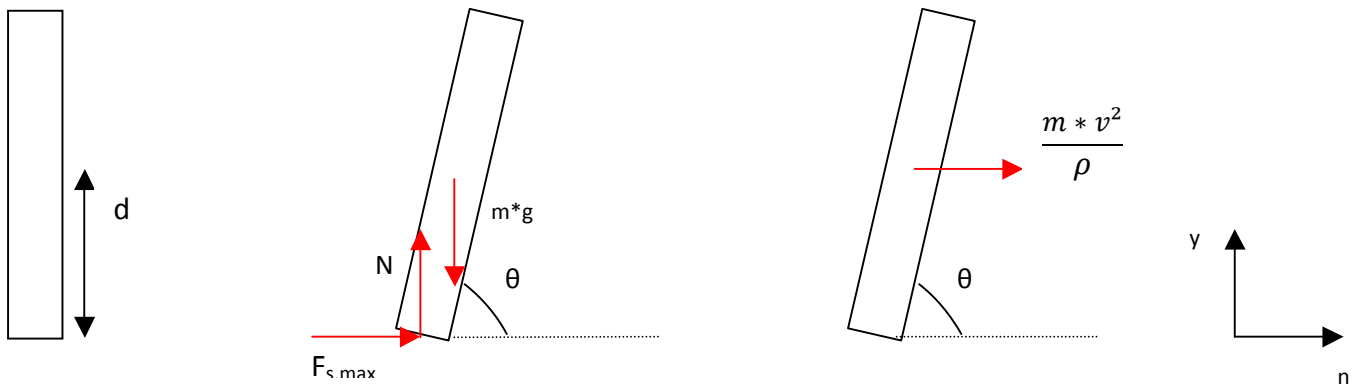
### Given:

A cyclist is riding at a speed of 50 km/h. He arrives at a crossroad and needs to turn left. The radius of the turn is 10 m. What is the necessary inclination angle? Does he have to reduce his speed to make a safe turn? What is the maximum possible speed?

Mass of the cyclist: 60 kg; mass of the bicycle: 12 kg; distance between ground and center of Gravity: 1,5 m (when he is riding vertically); Static coefficient of friction between wheels and ground: 0,3.

Asked:  $v_{\max}=?$ ,  $\theta=?$

### Solution:



$\Sigma F_y = \dots$

$$-m * g + N = 0 \Leftrightarrow N = m * g$$

$$\Leftrightarrow N = (60\text{kg} + 12\text{kg}) * 9,81 \text{ Newton/Kg} = 706,32 \text{ Newton}$$

$$F_{s, \max} = \mu_s * N \Leftrightarrow F_{s, \max} = \mu_s * m * g$$

$$\Leftrightarrow F_{s, \max} = 0,3 * 72\text{kg} * 9,81 \frac{\text{Newton}}{\text{Kg}} = 211,896 \text{ Newton}$$

$\Sigma F_n = \dots$

$$F_{s, \max} = (m * v^2) / \rho \text{ with } F_{s, \max} = \mu_s * m * g$$

$$\Leftrightarrow v = \sqrt{\mu_s * \rho * g} = \sqrt{0,3 * 10 \text{ m} * 9,81 \text{ N/kg}} = 5,424 \text{ m/s.}$$

$\Sigma M_A = \dots$

$$m * g * d * \cos(\theta) = d * \sin(\theta) * (m * v^2) / \rho \Leftrightarrow \cancel{m * g * d * \cos(\theta)} = \cancel{d * \sin(\theta) * m * g * \mu_s}$$

$$\Leftrightarrow \tan(\theta) = \frac{1}{\mu_s} = 3,33$$

$$\Leftrightarrow \theta = 73,28^\circ$$